

# Non-extensivity of hadronic systems.

L. Marques, E. Andrade-II and A. Deppman

Instituto de Física, Universidade de São Paulo - IFUSP, Rua do Matão, Travessa R 187, 05508-900 São Paulo-SP, Brazil

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## Abstract

The predictions from a non-extensive self-consistent theory recently proposed are investigated. Transverse momentum ( $p_T$ ) distribution for several hadrons obtained in  $p + p$  collisions are analyzed to verify if there are evidence for a limiting effective temperature and a limiting entropic factor. In addition, the hadron-mass spectrum proposed in that theory is confronted with available data.

It turns out that all  $p_T$ -distributions and the mass spectrum obtained in the theory are in good agreement with experiment with constant effective temperature and constant entropic factor. The results confirm that the non-extensive statistics plays an important role in the description of the thermodynamics of hadronic systems, and also that the self-consistent principle holds for energies as high as those achieved in the LHC. A discussion on the best  $p_T$ -distribution formula for fitting experimental data is presented.

## 1 Introduction

The Hagedorn's bootstrap idea based on a self-consistency requirement for the thermodynamics of fireballs predicted a limiting temperature for hadronic systems, and also gave formulas for transverse momentum ( $p_T$ ) distributions of secondaries and for the hadron-mass spectrum [1].

Experiments with  $\sqrt{s} > 10$  GeV, however, have shown that the  $p_T$ -distribution formula fails to describe the data. An empirical formula was proposed [2] including non-extensive statistics. It results that the modified formula can fit all available data for  $p_T$ -distributions.

Recently a non-extensive version of the self-consistency principle was proposed [3], leading to new formulas for mass spectrum and for transverse momentum distribution. The last one is similar to that proposed in Ref. [2]. In addition, the theory predicts a limiting effective temperature and a limiting entropic factor for all hadronic systems.

Table 1: Set of experimental data for  $p + p$  collisions.

Experiment	Particle	Reference
ALICE (LHC)	$\pi^0, \eta$	[4]
ALICE (LHC)	$\phi, \omega$	[5]
ALICE (LHC)	$\pi^\pm, P^\pm, K^\pm$	[6]
ATLAS (LHC)	$J/\psi$	[7]
CMS (LHC)	$J/\psi$	[8]
CMS (LHC)	$\Lambda_b^0$	[9]
CMS (LHC)	$K_S^0, \Lambda, \Xi^-$	[10]
LHCb (LHC)	$B^+$	[11]
LHCb (LHC)	$\phi$	[12]
LHCb (LHC)	$J/\psi$	[13]

In this work experimental data for  $p_T$ -distributions from different experiments and for several hadrons produced in  $p + p$  collisions at ultrarelativistic energies are analysed in order to investigate the theoretical predictions given in Ref. [3]. Also, the theoretical mass spectrum is compared to experimental data. The experimental data used in the present analysis are summarized in Table 1.

Initially it is important to clarify that the  $p_T$ -distribution given by

$$\frac{d^2 N}{dp_T dy} = gV \frac{p_T m_T \cosh y}{(2\pi)^2} \left( 1 + (q-1) \frac{m_T \cosh y - \mu}{T} \right)^{-\frac{q}{q-1}} \quad (1)$$

can be directly obtained from the Tsallis entropy[14] through the usual thermodynamical relations [15]. Despite this fact, many analysis apply other  $p_T$ -distributions [4, 5, 6, 9], as

$$\frac{d^2 N}{dp_T dy} = p_T \frac{dN}{dy} \frac{(n-1)(n-1)}{nC[nC + m_o(n-2)]} \left( 1 + \frac{m_T - m_o}{nC} \right)^{-n}. \quad (2)$$

In the equations above,  $y$  is the hadron rapidity,  $\mu$  is the chemical potential,  $m_T = \sqrt{p_T^2 + m^2}$ , with  $m$  being the hadron mass,  $n$  and  $C$  are constants,  $V$  is the volume and  $g$  is the degeneracy factor[15].

Although both formulas can be made quite similar by adopting

$$n = \frac{q}{q-1} \quad (3)$$

and

$$nC = \frac{T}{q-1}, \quad (4)$$

the factor  $m_T$  present in Eq. 1 and absent in Eq. 2 is sufficient to produce very different values for the parameters  $T$  and  $q$  when those equations are used to fit experimental data, even if quite good fittings are obtained with both equations.

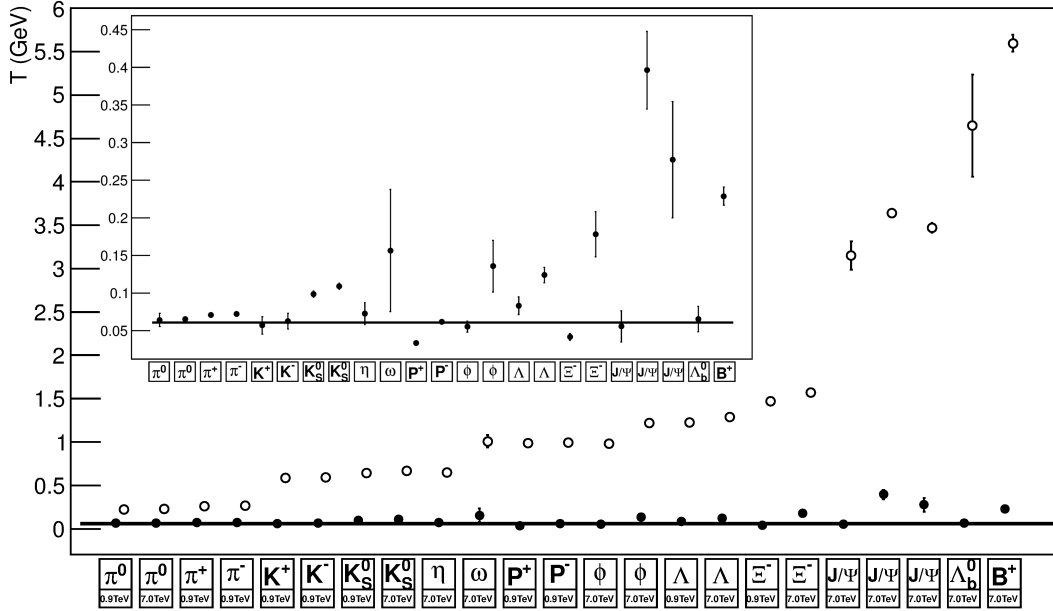


Figure 1: Effective temperature,  $T$ , resulting from the fittings of Eq. 1 (full symbols), assuming  $y = 0$  and  $\mu = 0$ , and Eq. 2 (open symbols). The inset shows the effective temperature obtained through the use of Eq. 1 in more details. Full lines indicate the constant value,  $T_o$ , which best fits the data obtained with Eq. 1

In Fig. 1 the effective temperatures obtained from those fittings are presented. It is clear that the temperature obtained with Eq. 2 varies in a broad range, systematically increasing with the hadron mass. The results obtained with Eq. 1, on the other hand, give temperatures spread over a much narrower range around a constant value  $T_o = (60.7 \pm 0.5)$  MeV (full lines in Fig. 1).

Comparing Eqs 1 and 2, it is easy to understand that the absence of the  $m_T$  factor in the latter gives rise to the increasing temperature behaviour observed in Fig. 1. Indeed, the effects of the increase in  $m_T$  due to the increase of  $p_T$  in Eq. 1 are reproduced in Eq. 2 by an increase of  $T$ .

The results of the entropic factor obtained from the fittings are plotted in Fig. 2. In this case, for both equations the results are spread around an average value with no evidence of a systematic trend. But again the values are spread over a broader range in the case of Eq. 2, while they are limited to a narrower range around  $q_o = 1.138 \pm 0.006$  when Eq. 1 is used.

In principle, there is nothing wrong with an increasing temperature for increasing hadron mass. The relevant points here are:

- i) The  $p_T$ -distribution obtained from Tsallis entropy by using the usual thermodynamical relations is Eq. 1, not Eq. 2.
- ii) If Tsallis statistics is the basis for a thermodynamical description of hadronic systems, then the self-consistency principle lead to a limiting effective temperature. Such limiting temperature is observed when Eq. 1 is used, as shown by the results in Fig. 1.
- iii) The theory predicts a limiting entropic factor, which is also observed in the analysis of  $p_T$ -distribution, as shown in Fig. 2.

These results are in agreement with recent analysis performed in Refs. [15, 16], where constant temperature and entropic factor were found with values similar to those obtained here, and extend those analysis by considering identified particles and by including  $p + p$  collisions up to  $\sqrt{s}=7$  GeV. Therefore, there are strong evidences that the non-extensive statistics plays an important role in the thermodynamical description of hadronic systems, and that the self-consistency conditions find support in the experimental data from ultrarelativistic collisions. It is worthwhile to stress the importance of using a  $p_T$ -distribution that is consistently derived from Tsallis entropy by the use of thermodynamical relations.

A crucial verification of the theory is related to the mass spectrum. In fact, if Hagedorn's theory fails to describe  $p_T$ -distributions for  $\sqrt{s} > 10$  GeV, it also has problems to describe the data for hadron-mass spectrum. The Hagedorn temperature,  $T_H$ , varies from 141 MeV up to 340 MeV, depending on the parametrization used for the mass spectrum formula, specially for the multiplying factor[17, 18, 19, 20, 21, 22, 23, 24, 25] . For the most used parametrization, however,  $T_H$  is much higher than that expected from hadron-hadron collisions, where  $T_H \approx 160$  MeV[17, 20].

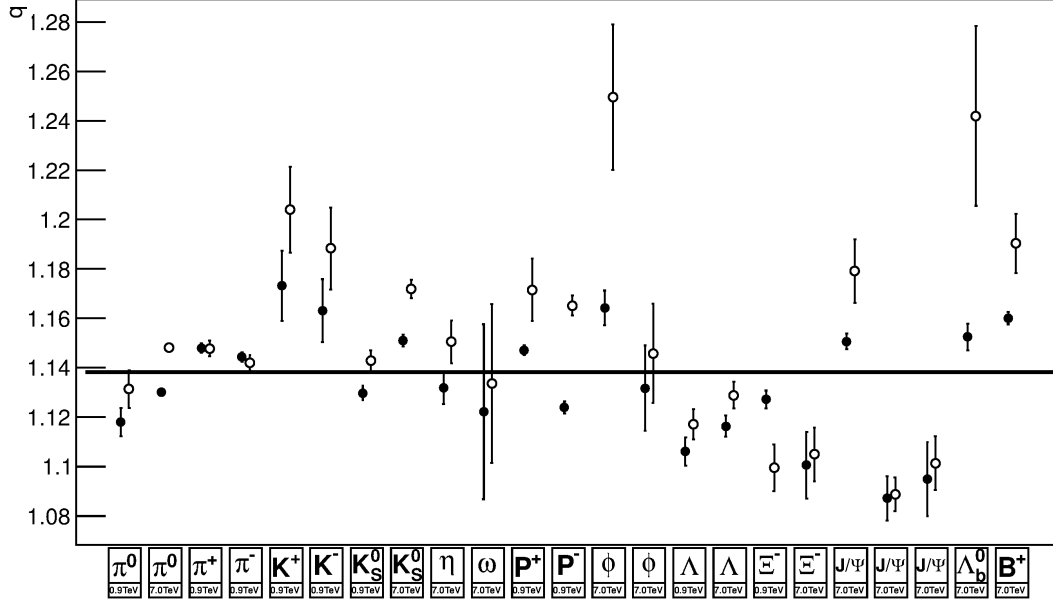


Figure 2: Entropic factor,  $q$ , resulting from the fittings of Eq. 1 (full symbols) and Eq. 2 (open symbols). The full line indicates a constant  $q$  fitted to the data obtained with Eq. 1.

According to the non-extensive self-consistent theory, the hadron-mass spectrum is given by

$$\rho(m) = \gamma m^{-5/2} e_q^{\beta_o m}, \quad (5)$$

where  $e_q^x$  is the  $q$ -exponential function [3] given by

$$e_q^x = [1 + (q - 1)x]^{1/(q-1)}. \quad (6)$$

It is important, therefore, to verify if this equation can describe the mass spectrum data with the same values  $T_o$  and  $q_o$  obtained in  $p + p$  collisions.

The cumulative hadron-mass distribution is given by

$$r(m) = \int \rho(m) dm = \frac{-2\gamma}{3} m^{-3/2} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{q-1}; -\frac{1}{2}; -(q-1)\beta m\right) + k, \quad (7)$$

where  $k$  is a constant and  ${}_2F_1(a, b; c; z)$  is the hypergeometric function. This equation was fitted to the available data for cumulative mass spectrum[17].

In Fig. 3 the best fitted curve is shown, and it is possible to observe a good agreement between data and calculation. The fitting procedure does not take into account data above 2 GeV, since the information above this

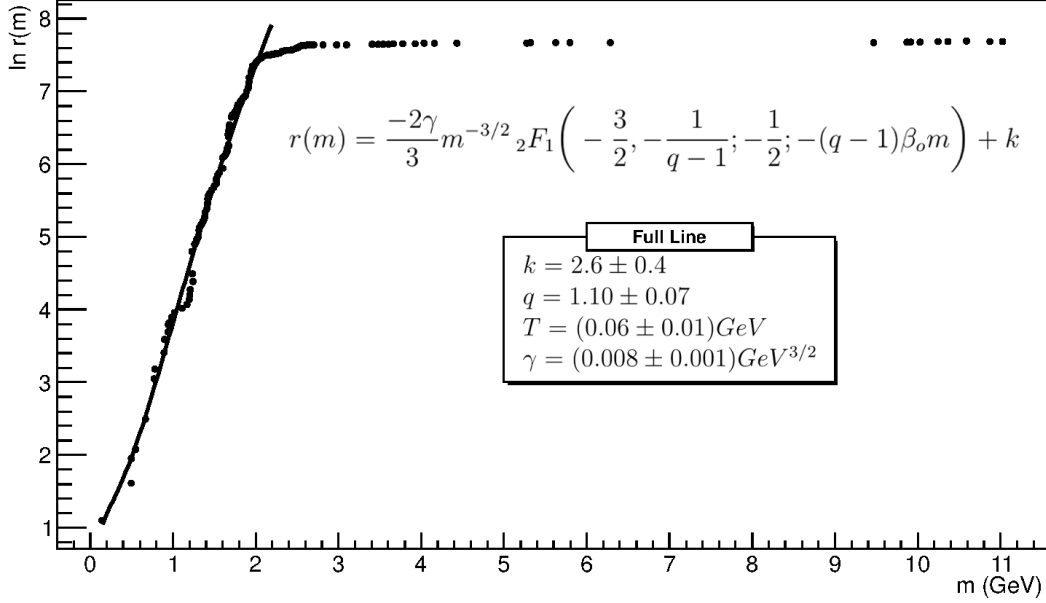


Figure 3: Cumulative hadron-mass spectrum. Full line represents the calculation with Eq. 7, and full-circles represent the available data taken from Ref. [17].

threshold is not considered reliable. This procedure is usual in the study of mass spectrum [17, 20].

The curve in Fig. 3 is obtained from Eq. 7 with  $T = (60 \pm 10)$  MeV and  $q = 1.10 \pm 0.07$ . Comparing these values with those obtained in  $p + p$  analysis it is clear that a good agreement is found between the results from  $p_T$ -distribution analysis and those from mass spectrum analysis. These results show that the non-extensive self-consistent theory proposed in Ref [3] can describe simultaneously the  $p_T$ -distribution and the hadron-mass spectrum with constant effective temperature and constant entropic factor.

A final remark can be made about the values  $T_o$  and  $q_o$ . According to the interpretation of non-extensivity given in Ref. [26],  $T_o$  and  $q_o$  are related to the critical temperature by

$$T_o = T_H + (q_o - 1) c, \quad (8)$$

where  $c$  is a constant depending on the energy transfer between the source and its surroundings and on thermodynamical properties of the medium [27]. In Ref. [16] it was shown that  $T_H = (192 \pm 15)$  MeV and  $c = -(950 \pm 10)$  MeV. It is interesting to observe that  $T_H$  is in good agreement with the critical temperature from lattice QCD [28, 29]. It is also clear that the values found here for  $T_o$  and  $q_o$  satisfy Eq. 8.

In conclusion, this work presents an extensive analysis of  $p_T$ -distribution from  $p+p$  collisions at ultrarelativistic energies in order to test the predictions of the non-extensive self-consistent theory proposed in Ref. [3]. The results show a limiting effective temperature  $T_o = (60.7 \pm 0.5)$  MeV and a limiting entropic factor  $q_o = 1.138 \pm 0.006$ .

Also the theoretical mass spectrum is compared with the available data resulting in good agreement between calculation and data for  $T = (60 \pm 10)$  MeV and  $q = 1.10 \pm 0.07$ . These values are in good agreement with the values  $T_o$  and  $q_o$  found in  $p_T$ -distribution analysis.

Finally, the relation between  $T_o$ ,  $q_o$  and  $T_H$  shows that the results obtained in the present work are in agreement with the critical temperature predicted in lattice QCD calculations.

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